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and target setting under DEA control

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**CORE**

DISCUSSION PAPER

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### **Centralized resource reduction and target setting under DEA control**

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#### **Abstract**

Data envelopment analysis (DEA) is a powerful tool for measuring the relative efficiencies of a set of decision making units (DMUs) such as schools and bank branches that transform multiple inputs to multiple outputs. In centralized decision-making systems, management normally imposes common resource constraints such as fixed capital, budgets for operating capital and staff count. In consequence, the profit or net value added of the units subject to resource reductions will decrease. In terms of performance evaluation combined with resource allocation, the interest of central management is to restore the general efficiency value of the DMUs. The paper makes four contributions to the literature: (1) we take into consideration the performance evaluation of the centralized budgeting of hierarchical organizations along with sales and market allocation within manufacturing and distribution organizations; (2) we address the evaluation problems that the central decision maker does not desire to deteriorate the efficiency score of the DMUs after input and/or output reduction; (3) we develop a common set of weights (CSW) method based on the goal program (GP) concept to control the total weight flexibility in the conventional DEA models; (4) we extend a new approach to optimize the inputs and/or outputs contraction such that the efficiency of all DMUs will get bigger than or equal to the efficiency of previous change. We ultimately present a numerical example involving with three inputs and two outputs to illustrate the applicability and efficacy of the proposed approach.

**Keywords:** data envelopment analysis, inputs and outputs deterioration, common set of weights.

**JEL Classification:** C14, M11, C61

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## 1. Introduction

Non-parametric frontier analysis was first introduced by Farrell (1957) and later developed as data envelopment analysis (DEA) by Charnes et al. (1978) into a linear programming based technique for efficiency assessment and ranking of decision making units (DMUs). DEA is a rapidly growing area of operational research that deals with the performance assessment of organizations (cf. Emrouznejad et al. (2008)).

Whereas the conventional analysis implicitly assumes that all DMU enjoy complete autonomy in their actions and access to free resource and product markets, performance analysis is increasingly used within organizations under a common management. A principal difference with respect to the prior assumptions is then that the DMU are subject to common resource and market constraints, imposed by a central decision maker. Obvious examples are found in centralized budgeting of hierarchical organizations as well as sales and market allocation within manufacturing and distribution organizations. Hence, in many real-world problems we must consider significant change in input and output measures. However, the central decision maker does not desire to deteriorate the efficiency score of the DMUs after input and/or output reduction, unless either the management action is contrary to the organizational objectives, which is absurd, or the evaluation is unrelated to the managerial objectives, which would render it meaningless. Several researchers have applied the input and/or output deterioration to DEA models in the DEA literature. Activity planning in DEA was proposed by Banker et al. (1989), Bogetoft (1993, 1994, 2000) and Golany and Tamir (1995). In the literature, as far as we know Cook and Kress (1999) were the first to introduce the idea of resource allocation in DEA by characterizing an equitable way for allocation of shared costs. However, their approach cannot provide the cost allocation directly for DMUs. Jahanshahloo et al. (2004) first indicated the shortcoming of Cook and Kress (1999)'s approach. Then they applied a simple method to achieve a costs allocation without solving any linear program. Cook and Zhu (2005) also extended the method of Cook and Kress (1999) to direct cost allocation. Lin (2011) extended the method of Cook and Zhu (2005) for allocating fixed resources with some additional constraints. Athanassopoulos (1995) proposed a method for target setting and resource allocation in multi-level planning problems using goal programming and DEA. Similar to the framework presented by Athanassopoulos (1995), Athanassopoulos (1998) proposed a resource allocation model, called TARBA consisting of two steps: (1) determining of the optimal weights using a multiplier DEA model (2) defining feasible trade-offs in allocation. Athanassopoulos et al. (1999) applied maximum and minimum bounds on inputs for each individual DMU that had to be satisfied after reallocation. Ito et al. (1999) reallocated the management resources to provide the maximum outputs using the concept of production possibility set of DEA-BCC model. Yan et al. (2002) developed an inverse generalized DEA model and they then discussed the application of the extended model to resource reallocation problem. Cook and Zhu (2003) developed a DEA model for efficiency measurement of highway maintenance crews as maximum achievable by reduction in input without impacting the outputs from the process. Beasley (2003) used the concept of DEA to maximize the average of the

efficiency scores of the DMUs as well as allocating fixed costs and output targets by a non-linear program problem. Amirteimoori and Kordrostami (2005) modified the constraints of Beasley (2003)'s model to prevent infeasibility in many cases. Korhonen and Syrjänen (2004) developed a resource-allocation model for the centralized organizations using DEA and multiple-objective linear programming to find an equitable allocation plan. Jahanshahloo et al. (2005) presented a method for allocating a fixed output fairly among DMUs without solving any linear program. Amirteimoori and Shafiei (2006) proposed a DEA-based method for removing a fix resource from all DMUs in a fair way such that the efficiency of units before and after reduction remains unchanged. Li and Cui (2008) presented a resource allocation framework consisting of various returns to scale model, inverse DEA model, common weight analysis model, and extra resource allocation algorithm. Li et al. (2009) first considered the linkage between the efficiency scores and the cost allocation and they then developed a DEA approach to allocating the fixed cost between DMUs. Pachkova (2009) proposed a model based on DEA to reallocate inputs, where this model was trade-off between the maximum allowed reallocation cost and the highest possible summation of efficiency of all DMUs. Vaz et al. (2010) first assessed the efficiency of the retail stores with several selling sections in a network DEA model under VRS and showed how resources reallocation and target setting using the approach proposed by Färe et al. (1997) improve the efficiency scores. Based on the parallel DEA model introduced by Kao (2009), Bi et al. (2011) suggested resource allocation and target setting for parallel production system. Amirteimoori and Mohaghegh Tabar (2010) proposed a DEA approach for resource allocation and target setting problems. In their setting, the decision maker desires to add a fixed additional resource equitably to all DMUs and demands a fixed additional output to distribute among the DMUs. Further, Amirteimoori and Emrouznejad (2010) presented a DEA-based approach to determine the highest possible input reduction and lowest possible output deterioration without reducing the efficiency score for each DMU. Recently, Lozano et al. (2011) introduced a number of non-radial, output-oriented and centralized DEA models for resource allocation and target setting for inputs with integer constraints. In the original DEA model, Charnes et al. (1978) proposed that the efficiency of a DMU can be obtained as the maximum of a ratio of weighted outputs to weighted inputs, subject to the condition that the same ratio for all DMUs must be less than or equal to one. In fact, there are no restrictions on how much weight (multiplier) can be placed on each input or output relative to the others. Thus, the endogenous weights for each individual DMU are chosen uniquely with its own efficiency in mind. This characteristic of DEA is called the "total weights flexibility". Obviously, it is possible that a particular DMU only takes into account weights on a few variables. Moreover, in the setting with a central decision maker evaluating a set of structurally comparable units in e.g. an organization or sector, it is highly implausible and overly conservative to assume that each DMU faces unique marginal costs and benefits for the common technology. Consequently, many applications involve decision makers providing a priori preference value for inputs and outputs weights to be taken into account in the efficiency evaluation.

Many researchers have focused on dealing with the problem of unacceptable weighting schemes. Dyson and Thanassoulis (1988) give another approach to absolute multiplier restrictions. Charnes et al. (1990) demonstrated that undesirable weighting plans are unavoidable results in many DEA applications. They thus proposed cone ratio restrictions models to provide more realistic weights. Thompson et al. (1995) modified models developed by Charnes et al. (1990). The “assurance region” introduced by Thompson et al. (1988, 1990) is a special case of the cone ratio concept. There are some extensions on the assurance region concept in the DEA literature such as Allen et al. (1997), Thanassoulis et al. (1998) and Cook and Zhu (2008). Bessent et al. (1988) presented the constrained facet analysis to deal with the inherent problem involving the occurrence of zero weights. Lang et al. (1995) improved this latter approach by adopting a two-stage approach. Similar methods have been suggested by Green et al. (1996) and Olesen and Petersen (1996). The common weights approach in DEA was initially introduced by Cook et al. (1990) and developed by Roll et al. (1991). Hosseinzadeh Lotfi et al. (2000) and Jahanshahloo et al. (2005) used a multi-objective problem to specify a common set of weights (CSW) for all DMUs using a non-linear transformation. A game-theoretical approach to CSW in a setting where the DMUs must agree upon a common endogenous evaluation is found in (Agrell and Bogetoft, 2010). In the recent study, Saati et al. (2012) proposed a two-phase CSW approach using an ideal virtual unit that is computationally efficient. Their method was applied in energy regulation using panel data from 286 Danish district heating plants.

In this paper, we propose an alternative DEA-based method for a centrally imposed resource or output reduction across the reference set. In other words, this study addresses the following question: how much should the inputs and outputs for each DMU is reduced subject to the conditions that the efficiency scores of all DMUs increase? Consistent with the setting for a central evaluator, we use the DEA-based method in order to get better efficiency scores for all DMUs after the reduction amount of inputs and outputs.

The rest of this paper is organized into five sections. In Section 2, we present a brief review of the conventional DEA model and in Section 3 we propose the common-weights DEA model. Section 4 presents the details of the model proposed in this study. In Section 5 we show a numerical example to illustrate the efficacy of the proposed method. In Section 6, we close the paper with conclusions and future research directions.

## 2. The traditional DEA model

Data envelopment analysis (DEA) is essentially estimating a convex hull covering a set of decision making units (DMU) and radially projecting them against the hull in a specified direction.

Suppose that there are  $n$  DMUs to be evaluated where every  $DMU_j$ ,  $j=1, \dots, n$ , produces  $s$  outputs  $y_{rj} \in R^+$ ,  $r=1, \dots, s$ , using the  $m$  inputs,  $x_{ij} \in R^+$ ,  $i=1, \dots, m$ . The input-oriented model (CCR or CRS for constant returns to scale) for evaluating the relative efficiency of a given  $DMU_o$  is as follows (Charnes et al. 1978):

$$\begin{aligned}
& \max \frac{\sum_{r=1}^s w_r y_{ro}}{\sum_{i=1}^m \mu_i x_{io}} \\
& \text{s.t.} \quad \frac{\sum_{r=1}^s w_r y_{rj}}{\sum_{i=1}^m \mu_i x_{ij}} \leq 1, \quad j = 1, \dots, n, \\
& \quad w_r, \mu_i \geq \varepsilon, \quad r = 1, \dots, s; \quad i = 1, \dots, m.
\end{aligned} \tag{1}$$

where  $\varepsilon$  is a positive non-Archimedean infinitesimal number. The model (1) is also called the multiplier model. It is clear that the model (1) is non-linear and it can be converted to the linear program problem via Charnes and Cooper (1962)'s method as shown in model (2).

$$\begin{aligned}
& \bar{\theta}_o^* = \max \sum_{r=1}^s u_r y_{ro} \\
& \text{s.t.} \quad \sum_{i=1}^m v_i x_{io} = 1, \\
& \quad \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad j = 1, \dots, n, \\
& \quad u_r, v_i \geq \varepsilon, \quad r = 1, \dots, s; \quad i = 1, \dots, m.
\end{aligned} \tag{2}$$

**Definition 1:**  $DMU_o$  is efficient if there exists at least one optimal  $(u^*, v^*)$  of model (2) with  $u^* \geq \varepsilon$ ,  $v^* \geq \varepsilon$  and  $\sum_{r=1}^s u_r^* y_{ro} = 1$ . Otherwise,  $DMU_o$  is inefficient.

### 3. The common-weights DEA model

The relative efficiency using the multiplier DEA model is determined by assigning weights to the inputs and outputs of DMU to maximize the ratio of the weighted sum of outputs to the weighted sum of inputs. The only underlying assumption for the weights on inputs and outputs is non-negativity (called “total weights flexibility”). The calculation of DEA scores requires a linear program (2) per DMU and obtains an individual set of endogenous weights. We recall that the differences among the individual weights may be unacceptable for management reasons, market reasons or by technical or economic necessity. To cope with these difficulties, the common set of weights (CSW) model can be used to generate a common set of weights for all DMUs which are able to produce the highest efficiency score at the same time. In the ensuing section, we examine a CSW model based on the multi-objective program (MOP). Many researchers have investigated the relationships between DEA and MOP from the different aspects (e.g., see Hosseinzadeh Lotfi et al. 2010a, 2010b; Yang et al. 2009). To pursue our aim, we can

equivalently consider the following multi-objective fractional program (MOFP) for measuring the efficiency of all DMUs simultaneously:

$$\begin{aligned} \theta^* = \max \left\{ \frac{\sum_{r=1}^s u_r y_{r1}}{\sum_{i=1}^m v_i x_{i1}}, \frac{\sum_{r=1}^s u_r y_{r2}}{\sum_{i=1}^m v_i x_{i2}}, \dots, \frac{\sum_{r=1}^s u_r y_{rm}}{\sum_{i=1}^m v_i x_{im}} \right\} \\ \text{s.t.} \quad \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \leq 1, \quad j = 1, \dots, n, \\ u_r, v_i \geq \varepsilon, \quad r = 1, \dots, s; \quad i = 1, \dots, m. \end{aligned} \quad (3)$$

Since over two decades ago, many methods for solving multi-objective problems have been developed in the optimization literature (see e.g., Hwang and Masud 1979; Steuer 1986). In this paper, we adapt the MOFP model (3) to a goal program (GP) model which was developed by Freed and Glover (1981) and extended in (Freed and Glover 1986; Glover 1990). In the maximization MOFP model (3),  $\sum_{r=1}^s u_r y_{rj} / \sum_{i=1}^m v_i x_{ij}$  is the  $j^{\text{th}}$  objective function which, based on the constraints, should be as close as possible to efficiency score unity. In other words, the goal or aspiration level for each objective function in model (3) is to take the unity value or full technical efficiency. The difference between  $j^{\text{th}}$  objective function and its goal is defined as the negative and positive deviations, denoted by  $s_j^-$  ( $j=1, \dots, n$ ) and  $s_j^+$  ( $j=1, \dots, n$ ) respectively. The purpose of the GP method is to minimize these deviations from the preset goals for each objective function. There exist some methods to define the objective function  $f(s^-, s^+)$ , where each of them leads to different GP methods. We use weighted GP, which minimizes the weighted sum of the deviational variables as  $f(s^-, s^+) = \sum_{j=1}^n (\alpha_j s_j^- + \beta_j s_j^+)$  where  $\alpha_j$  and  $\beta_j$  present the weights of the negative and positive deviations on the  $j^{\text{th}}$  objective function and are characterized by the decision maker. Without loss of generality, we assume the identical deviations weights for all DMUs by considering  $\alpha_j = \beta_j = 1$ . To close the gap between the value of each objective function and the efficiency score one, we can consider three cases: (1)  $\sum_{r=1}^s u_r y_{rj} / \sum_{i=1}^m v_i x_{ij} \leq 1$ , using the negative deviational variable  $s_j^-$ ; (2)  $\sum_{r=1}^s u_r y_{rj} / \sum_{i=1}^m v_i x_{ij} \geq 1$ , using the positive deviational variable  $s_j^+$ ; (3)  $\sum_{r=1}^s u_r y_{rj} / \sum_{i=1}^m v_i x_{ij} \leq 1$ , and  $\sum_{r=1}^s u_r y_{rj} / \sum_{i=1}^m v_i x_{ij} \geq 1$ , at the same time using the negative and positive deviational variables. This representation thus results in the following model with a single objective function:

$$\begin{aligned}
\min \quad & \sum_{j=1}^n (s_j^- + s_j^+) \\
s.t. \quad & \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} + s_j^- - s_j^+ = 1, \quad j = 1, \dots, n, \\
& \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \leq 1, \quad j = 1, \dots, n, \\
& s_j^- \geq 0, \quad s_j^+ \geq 0, \quad j = 1, \dots, n, \\
& u_r, \quad v_i \geq \varepsilon, \quad r = 1, \dots, s; \quad i = 1, \dots, m.
\end{aligned} \tag{4}$$

Notice that  $s_j^+$  ( $j = 1, \dots, n$ ) in model (4) is not allowed to take the positive value since the positive value of  $s_j^+$  ( $j = 1, \dots, n$ ) and the zero value of  $s_j^-$  ( $j = 1, \dots, n$ ) does not satisfy the second set of constraints. We can thus omit  $s_j^+$  in model (4) and consequently the second set of constraints is redundant. Model (4) is a non-linear program and its purpose is to minimize the total gaps to reach goal. Based on the GP concept, we present a new linear programming model for solving the MOFP model (3). In DEA, every DMU can minimize the sum of the total virtual gaps to receive the benchmarking frontier by adding  $s_j^+$  to  $\sum_{r=1}^s u_r y_{rj}$  and taking  $s_j^-$  away from  $\sum_{i=1}^m v_i x_{ij}$ . As a result, the multi-objective fractional program (3) can be converted to the following linear model:

$$\begin{aligned}
\min \quad & \sum_{j=1}^n (s_j^- + s_j^+) \\
s.t. \quad & \frac{\sum_{r=1}^s u_r y_{rj} + s_j^+}{\sum_{i=1}^m v_i x_{ij} - s_j^-} = 1, \quad j = 1, \dots, n, \\
& \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \leq 1, \quad j = 1, \dots, n, \\
& s_j^- \geq 0, \quad s_j^+ \geq 0, \quad j = 1, \dots, n, \\
& u_r, \quad v_i \geq \varepsilon, \quad r = 1, \dots, s; \quad i = 1, \dots, m.
\end{aligned} \tag{5}$$

where the non-linear program (5) can be simply changed to the following linear program:



$$\begin{aligned}
\min \quad & \sum_{j=1}^n (s_j^- + s_j^+) \\
s.t. \quad & \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} + s_j^- + s_j^+ = 0, \quad j = 1, \dots, n, \\
& \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad j = 1, \dots, n, \\
& s_j^- \geq 0, \quad s_j^+ \geq 0, \quad j = 1, \dots, n, \\
& u_r, v_i \geq \varepsilon, \quad r = 1, \dots, s, \quad i = 1, \dots, m.
\end{aligned} \tag{6}$$

We further simplify the program by replacing  $s_j^- + s_j^+$  to  $s_j$ :

$$\begin{aligned}
\min \quad & \sum_{j=1}^n s_j \\
s.t. \quad & \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} + s_j = 0, \quad j = 1, \dots, n, \quad (7a) \\
& \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad j = 1, \dots, n, \quad (7b) \\
& s_j \geq 0, \quad j = 1, \dots, n, \\
& u_r, v_i \geq \varepsilon, \quad r = 1, \dots, s; \quad i = 1, \dots, m.
\end{aligned} \tag{7}$$

From constraint (7a),  $\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} = -s_j$ . If we substitute the left hand side of constraint (7b) with  $-s_j$ , we obtain  $-s_j \leq 0$  equivalently  $s_j \geq 0$ . It shows that the constraints (7b) are redundant and consequently the second constraints in the models (5) and (6) are redundant and can be omitted from model.

Using the optimal solutions  $(u_r^*, v_i^*, s_j^*) \forall r, i, j$  to (7), the efficiency scores for  $DMU_j$ ,  $j = 1, \dots, n$ , are calculated as follows:

$$\theta_j^* = \frac{\sum_{r=1}^s u_r^* y_{rj}}{\sum_{i=1}^m v_i^* x_{ij}} = 1 - \frac{s_j^*}{\sum_{i=1}^m v_i^* x_{ij}}, \quad j = 1, \dots, n. \tag{8}$$

**Definition 2:**  $DMU_j$ ,  $j = 1, \dots, n$ , is non-dominated if and only if  $s_j^* = 0$ ,  $j = 1, \dots, n$ , in the model (7). In other words,  $DMU_j$  is non-dominated if and only if  $\theta_j^* = 1$ , in equation (8).

#### 4. A proposed method

In performance evaluation of centralized organizations with some common control, the interest to maintain optimal technical efficiency goes hand in hand with the necessity to comply with resource and market constraints. For instance, a public authority is staffing and supplying schools with special resources, as well as assigning students to regions and

districts. A budget reduction to the sector must be implemented across the schools, as well as demographic changes may lead to reductions in the number of students both admitted and graduated. In both cases, it is primordial for the central manager to maintain or improve technical efficiency or its units after the resource reallocation. In this section, we propose an alternative data-based approach for determining the highest possible input reduction and the lowest possible output deterioration without reducing the efficiency score for each DMU derived from CSW approach.

Recall that we consider a technology with  $m$  inputs,  $x_{ij} \in R^+$ ,  $i = 1, \dots, m$ , and  $s$  outputs,  $y_{rj} \in R^+$ ,  $r = 1, \dots, s$ . Assume that  $I_1 = \{i_1, i_2, \dots, i_k\}$ ,  $I_2 = \{1, 2, \dots, m\} - I_1$ ,  $O_1 = \{r_1, r_2, \dots, r_t\}$  and  $O_2 = \{1, 2, \dots, s\} - O_1$ , where  $I_1$  and  $O_1$  are the subset of inputs and outputs, respectively, which the organization is willing to reduce these inputs and outputs. The total reduction of the  $i^{th}$  input and the  $r^{th}$  output, denoted by  $C_i$ ,  $i \in I_1$ , and  $P_r$ ,  $r \in O_1$ , can be obtained as:

$$\sum_{j=1}^n \bar{c}_{ij} = C_i, \quad i \in I_1,$$

$$\sum_{j=1}^n \bar{p}_{rj} = P_r, \quad r \in O_1.$$

where  $\bar{c}_{ij}$  and  $\bar{p}_{rj}$  are, respectively, the  $i^{th}$  reduced input and the  $r^{th}$  reduced output with respect to  $j^{th}$  DMU. Let us  $\theta_j^*$  be the efficiency score of  $j^{th}$  DMU obtained from (8) without changing the data. In order to determine the adequate assigned values to  $\bar{c}_{ij}$  and  $\bar{p}_{rj}$  and keep efficiency scores greater than or equal to  $\theta_j^*$  for  $DMU_j$ , we require to consider the following set of constraints:

$$(9i) \quad \frac{\sum_{r \in O_2} u_r y_{rj} + \sum_{r \in O_1} u_r (y_{rj} - \bar{p}_{rj})}{\sum_{i \in I_2} v_i x_{ij} + \sum_{i \in I_1} v_i (x_{ij} - \bar{c}_{ij})} \geq \theta_j^*, \quad j = 1, \dots, n,$$

$$(9ii) \quad \frac{\sum_{r \in O_2} u_r y_{rj} + \sum_{r \in O_1} u_r (y_{rj} - \bar{p}_{rj})}{\sum_{i \in I_2} v_i x_{ij} + \sum_{i \in I_1} v_i (x_{ij} - \bar{c}_{ij})} \leq 1, \quad j = 1, \dots, n,$$

$$(9iii) \quad \sum_{j=1}^n \bar{c}_{ij} = C_i, \quad i \in I_1, \tag{9}$$

$$(9iv) \quad \sum_{j=1}^n \bar{p}_{rj} = P_r, \quad r \in O_1,$$

$$(9v) \quad \bar{c}_{ij} \leq x_{ij}, \quad i \in I_1, \quad j = 1, \dots, n,$$

$$(9vi) \quad \bar{p}_{rj} \leq y_{rj}, \quad r \in O_1, \quad j = 1, \dots, n,$$

$$u_r, v_i \geq \varepsilon, \quad \bar{c}_{ij}, \bar{p}_{rj} \geq 0, \quad r = 1, \dots, s, i = 1, \dots, m, j = 1, \dots, n.$$

where  $\bar{c}_{ij}$  and  $\bar{p}_{rj}$  are decision variables in addition to  $u_r$  and  $v_i$ , thereby, (9) is a non-linear system in the presence of constraints (9i) and (9ii). The interpretation of each set of constraints in (9) is concisely expressed as

- *Constraints (9i)* ensure that the efficiency scores for each DMU are not smaller than the primary efficiency  $\theta_j^*$  calculated by (8).
- *Constraints (9ii)* imply that the efficiency scores for each DMU are less than or equal to unity.
- *Constraints (9iii)* set the sum of reduced inputs equal to  $C_i$ .
- *Constraints (9iv)* set the sum of reduced outputs equal to  $P_r$ .
- *Constraints (9v)* enforce the maximum contraction of  $i^{\text{th}}$  input is  $x_{ij}$  for  $j^{\text{th}}$  DMU.
- *Constraints (9vi)* enforce the maximum contraction of  $r^{\text{th}}$  output is  $y_{rj}$  for  $j^{\text{th}}$  DMU.

We can rewrite constraints (9i) and (9ii) as follows:

$$\frac{\sum_{r=1}^s u_r y_{rj} - \sum_{r \in O_1} u_r \bar{p}_{rj}}{\sum_{i=1}^m v_i x_{ij} - \sum_{i \in I_1} v_i \bar{c}_{ij}} \geq \theta_j^*, \quad j = 1, \dots, n, \quad \frac{\sum_{r=1}^s u_r y_{rj} - \sum_{r \in O_1} u_r \bar{p}_{rj}}{\sum_{i=1}^m v_i x_{ij} - \sum_{i \in I_1} v_i \bar{c}_{ij}} \leq 1, \quad j = 1, \dots, n. \quad (10)$$

Using alteration variables  $u_r \bar{p}_{rj} = p_{rj}$  and  $v_i \bar{c}_{ij} = c_{ij}$ , the non-linear system (9) can be transformed to the following linear system:

$$\begin{aligned} (11i) \quad & \frac{\sum_{r=1}^s u_r y_{rj} - \sum_{r \in O_1} p_{rj}}{\sum_{i=1}^m v_i x_{ij} - \sum_{i \in I_1} c_{ij}} \geq \theta_j^*, \quad j = 1, \dots, n, \\ (11ii) \quad & \frac{\sum_{r=1}^s u_r y_{rj} - \sum_{r \in O_1} p_{rj}}{\sum_{i=1}^m v_i x_{ij} - \sum_{i \in I_1} c_{ij}} \leq 1, \quad j = 1, \dots, n, \\ (11iii) \quad & \sum_{j=1}^n c_{ij} = v_i C_i, \quad i \in I_1, \\ (11iv) \quad & \sum_{j=1}^n p_{rj} = u_r P_r, \quad r \in O_1, \\ (11v) \quad & c_{ij} \leq v_i x_{ij}, \quad i \in I_1, \quad j = 1, \dots, n, \\ (11vi) \quad & p_{rj} \leq u_r y_{rj}, \quad r \in O_1, \quad j = 1, \dots, n, \\ & u_r, v_i \geq \varepsilon, \quad c_{ij}, p_{rj} \geq 0, \quad r = 1, \dots, s; i = 1, \dots, m; j = 1, \dots, n. \end{aligned} \quad (11)$$

At present, the aim is to solve the above system to determine the amount of reduction in the inputs and outputs. There exist some methods to solve system (11) such as Gauss-Jordan and Gaussian elimination methods (See Datta 1994 for further details).

Inputs and outputs contraction for each DMU must be proportional to the present inputs and outputs to ensure an equitable impact. To deal with this problem, we can take into

account the relative importance or weights of the inputs and outputs. There are various method for determination of weights such as eigenvector method, weighted square method and entropy method. In this paper, we use the following simple scaling formulas in order to make the model more reasonable:

$$\rho_{ij} = \frac{x_{ij}}{\sum_{i=1}^n x_{it}}, \quad j = 1, \dots, n, \quad i \in I_1, \quad \mu_{rj} = \frac{y_{rj}}{\sum_{r=1}^n y_{rt}}, \quad j = 1, \dots, n, \quad r \in O_1.$$

where  $\rho_{ij}$  and  $\mu_{rj}$  are input and output multipliers for the  $i^{\text{th}} \in I_1$  input and  $r^{\text{th}} \in O_1$  output, respectively, for each DMU. Note that in the above formulations  $\sum_{j=1}^n \rho_{ij} = \sum_{j=1}^n \mu_{rj} = 1$ . In the presence of the multipliers  $\rho_{ij}$  and  $\mu_{rj}$  system (11) might be infeasible, therefore, we are not able to solve (11) using common approaches. In this section, we develop a new method to solve (11) with respect to the GP concept. In constraint (11i) it is desirable that the efficiency score after input-output reductions will be greater than or equal to the efficiency score before incorporating changes into data. Moreover, we can prevent infeasibility from constraint (11iii) and (11iv) by defining the negative and positive deviational variables. We denote the negative and positive deviation variables by  $\alpha_{ij}^-$  and  $\alpha_{ij}^+$  for  $c_{ij}$  ( $i \in I_1$ ), and  $\beta_{rj}^-$  and  $\beta_{rj}^+$  for  $p_{rj}$  ( $r \in O_1$ ). Notice that under the input-output effects, the goals of  $c_{ij}$  in (11iii) and  $p_{rj}$  in (11iv) are  $v_i(\rho_{ij}C_i)$  and  $u_r(\mu_{rj}P_r)$ , respectively. According to GP concepts, we minimize the sum of the defined negative and positive deviational variables to achieve the goals. Thereupon, we create the following model:

$$\begin{aligned} \min \quad & \sum_{j=1}^n \sum_{i \in I_1} (\alpha_{ij}^- + \alpha_{ij}^+) + \sum_{j=1}^n \sum_{r \in O_1} (\beta_{rj}^- + \beta_{rj}^+) \\ \text{s.t.} \quad & \frac{\sum_{r=1}^s u_r y_{rj} - \sum_{r \in O_1} p_{rj}}{\sum_{i=1}^m v_i x_{ij} - \sum_{i \in I_1} c_{ij}} \geq \theta_j^*, \quad j = 1, \dots, n, \\ & \frac{\sum_{r=1}^s u_r y_{rj} - \sum_{r \in O_1} p_{rj}}{\sum_{i=1}^m v_i x_{ij} - \sum_{i \in I_1} c_{ij}} \leq 1, \quad j = 1, \dots, n, \\ & c_{ij} + \alpha_{ij}^- - \alpha_{ij}^+ = v_i(\rho_{ij}C_i), \quad i \in I_1, \quad j = 1, \dots, n, \\ & p_{rj} + \beta_{rj}^- - \beta_{rj}^+ = u_r(\mu_{rj}P_r), \quad r \in O_1, \quad j = 1, \dots, n, \\ & c_{ij} \leq v_i x_{ij}, \quad i \in I_1, \quad j = 1, \dots, n, \\ & p_{rj} \leq u_r y_{rj}, \quad r \in O_1, \quad j = 1, \dots, n, \\ & \sum_{j=1}^n c_{ij} = v_i C_i, \quad i \in I_1; \quad \sum_{j=1}^n p_{rj} = u_r P_r, \quad r \in O_1, \\ & u_r, v_i \geq \varepsilon, c_{ij}, p_{rj}, \alpha_{ij}^-, \alpha_{ij}^+, \beta_{rj}^-, \beta_{rj}^+ \geq 0, \quad r = 1, \dots, s; i = 1, \dots, m; j = 1, \dots, n. \end{aligned} \tag{12}$$

In the above model,  $\varepsilon$  is a non-Archimedean constant. Model (12) is a fractional program problem that cannot be solved by standard optimization methods. Hence, we simply convert (12) into the following linear program problem:

$$\begin{aligned}
\min \quad & \sum_{j=1}^n \sum_{i \in I_1} (\alpha_{ij}^- + \alpha_{ij}^+) + \sum_{j=1}^n \sum_{r \in O_1} (\beta_{rj}^- + \beta_{rj}^+) \\
s.t. \quad & \left( \sum_{r=1}^s u_r y_{rj} - \sum_{r \in O_1} p_{rj} \right) - \theta_j^* \left( \sum_{i=1}^m v_i x_{ij} - \sum_{i \in I_1} c_{ij} \right) \geq 0, \quad j = 1, \dots, n, \\
& \left( \sum_{r=1}^s u_r y_{rj} - \sum_{r \in O_1} p_{rj} \right) - \left( \sum_{i=1}^m v_i x_{ij} - \sum_{i \in I_1} c_{ij} \right) \leq 0, \quad j = 1, \dots, n, \\
& c_{ij} + \alpha_{ij}^- - \alpha_{ij}^+ = v_i (\rho_{ij} C_i), \quad i \in I_1, \quad j = 1, \dots, n, \\
& p_{rj} + \beta_{rj}^- - \beta_{rj}^+ = u_r (\mu_{rj} P_r), \quad r \in O_1, \quad j = 1, \dots, n, \\
& c_{ij} \leq v_i x_{ij}, \quad i \in I_1, \quad j = 1, \dots, n, \\
& p_{rj} \leq u_r y_{rj}, \quad r \in O_1, \quad j = 1, \dots, n, \\
& \sum_{j=1}^n c_{ij} = v_i C_i, \quad i \in I_1, \\
& \sum_{j=1}^n p_{rj} = u_r P_r, \quad r \in O_1, \\
& u_r, v_i \geq \varepsilon, c_{ij}, p_{rj}, \alpha_{ij}^-, \alpha_{ij}^+, \beta_{rj}^-, \beta_{rj}^+ \geq 0, \quad r = 1, \dots, s; i = 1, \dots, m; j = 1, \dots, n.
\end{aligned} \tag{13}$$

**Theorem:** There always exists a feasible solution to model (13).

**Proof.** We have the following feasible solution to (13):

$$\begin{aligned}
v_i &= \frac{\sum_{j=1}^n c_{ij}}{C_i} \quad i \in I_1; \quad u_r = \frac{\sum_{j=1}^n p_{rj}}{P_r}, \quad r \in O_1, \\
v_i &= 1, \quad i \in I_2, \quad u_r = \frac{\gamma_j}{k y_{rj}} \quad r \in O_2, j = 1, \dots, n
\end{aligned}$$

where  $\gamma_j = \theta_j^* \sum_{i \in I_2} x_{ij}$  and  $|O_2| = k$ . Note that  $|f|$  represents a cardinal number of a set  $f$ .

Therefore, we have

$$\sum_{r \in O_2} u_r y_{rj} = \sum_{r \in O_2} \frac{\gamma_j}{k y_{rj}} y_{rj} = \gamma_j, \quad j = 1, \dots, n$$

In addition, we have

$$\begin{aligned}
c_{ij} &= v_i x_{ij}, \quad i \in I_1, j = 1, \dots, n, \Rightarrow c_{ij} = \frac{x_{ij} \sum_{j=1}^n c_{ij}}{C_i} \quad i \in I_1, \quad j = 1, \dots, n \\
p_{rj} &= u_r y_{rj}, \quad r \in O_1, j = 1, \dots, n, \Rightarrow p_{rj} = \frac{y_{rj} \sum_{j=1}^n p_{rj}}{P_r} \quad i \in I_1, \quad j = 1, \dots, n
\end{aligned}$$

$$\begin{aligned}
\sum_{r \in O_1} u_r y_{rj} &= \sum_{r \in O_1} p_{rj} \\
\sum_{i \in I_1} v_i x_{ij} &= \sum_{i \in I_1} c_{ij} \\
\alpha_{ij}^- &= \begin{cases} v_i(\rho_{ij} C_i) - c_{ij} & v_i(\rho_{ij} C_i) \geq c_{ij} \\ 0 & v_i(\rho_{ij} C_i) \leq c_{ij} \end{cases} \quad \begin{matrix} i \in I_1, & j = 1, \dots, n, \\ i \in I_1, & j = 1, \dots, n, \end{matrix} \\
\alpha_{ij}^+ &= \begin{cases} c_{ij} - v_i(\rho_{ij} C_i) & v_i(\rho_{ij} C_i) \leq c_{ij} \\ 0 & v_i(\rho_{ij} C_i) \geq c_{ij} \end{cases} \quad \begin{matrix} i \in I_1, & j = 1, \dots, n, \\ i \in I_1, & j = 1, \dots, n, \end{matrix} \\
\beta_{rj}^- &= \begin{cases} u_r(\mu_{rj} P_r) - p_{rj} & u_r(\mu_{rj} P_r) \geq p_{rj} \\ 0 & u_r(\mu_{rj} P_r) \leq p_{rj} \end{cases} \quad \begin{matrix} r \in O_1, & j = 1, \dots, n, \\ r \in O_1, & j = 1, \dots, n, \end{matrix} \\
\beta_{rj}^+ &= \begin{cases} p_{rj} - u_r(\mu_{rj} P_r) & u_r(\mu_{rj} P_r) \leq p_{rj} \\ 0 & u_r(\mu_{rj} P_r) \geq p_{rj} \end{cases} \quad \begin{matrix} r \in O_1, & j = 1, \dots, n, \\ r \in O_1, & j = 1, \dots, n, \end{matrix}
\end{aligned}$$

The above determined feasible solution can be thus satisfied all the constraints of model (13). The proof is complete. ■

## 5. A numerical example

In this section, we use panel data from a banking application proposed by Kao and Hwang (2009) and also used by Amirteimoori and Emrouznejad (2010) to illustrate the applicability of the proposed model. To assess the impact of information technology (IT) on bank performance, we take into account three inputs and two outputs described below:

- The input 1 (I1): IT budget (USD)
- The input 2 (I2): Fixed assets (USD)
- The input 3 (I3): Staff (headcount)
- The output 1 (O1): Deposits (USD)
- The output 2 (O2): Profit (USD).

The inputs and outputs data for 27 banks in a period of 1987–1989 are reported in Table 1.

In the first step we apply the proposed model (7) to obtain the optimal common weights. Then we measure the efficiency score ( $\theta_j^*$ ) of the banks using the equation (8) reported in Table 2. In the second step the banking system was forced to reduce the IT budget and the profit values owing to some exogenous financial constraints. Therefore, the present budget, 5.8916 billion dollars, must be reduced by 3 billion dollars (i.e.,  $C_1 = 3$ ). In such case, management expects that the bank's profits will shrink from 11.948 billion dollars to 6.948 billion dollars (i.e.,  $P_2 = 5$ ). To determine the adequate values of IT budget-profit reductions (denoted by  $c_{1j} - p_{2j}$ ) for each bank, we first apply model (13) to get the optimal solutions of  $c_{1j}$  and  $p_{2j}$ ,  $j = 1, \dots, 27$ . We then use alteration variables  $u_r \bar{p}_{rj} = p_{rj}$  and  $v_i \bar{c}_{ij} = c_{ij}$  to obtain the amount of IT budget and profit reduction denoted by  $\bar{c}_{1j}$  and  $\bar{p}_{2j}$ ,  $j = 1, \dots, 27$ , respectively. The optimal values of  $\bar{c}_{1j}$  and  $\bar{p}_{2j}$  are presented in Table 2. In the last step, to re-gauge the efficiency scores ( $\theta_j^{*new}$ ) of the

banks we use the proposed common-weights DEA model (7) and equation (8) in the presence of the new values for IT budget and profit determined from the preceding step. The result is presented in Table 2.

Table 1. The input-output data for 27 banks

DMU <sub>j</sub>	$I_1$	$I_2$	$I_3$	$O_1$	$O_2$
1	0.150	0.713	13.300	14.478	0.232
2	0.170	1.071	16.900	19.502	0.340
3	0.235	1.224	24.000	20.952	0.363
4	0.211	0.363	15.600	13.902	0.211
5	0.133	0.409	18.485	15.206	0.237
6	0.497	5.846	56.420	81.186	1.103
7	0.060	0.918	56.420	81.186	1.103
8	0.071	1.235	12.000	11.441	0.199
9	1.500	18.12	89.510	124.072	1.858
10	0.120	1.821	19.800	17.425	0.274
11	0.120	1.915	19.800	17.425	0.274
12	0.050	0.874	13.100	14.342	0.177
13	0.370	6.918	12.500	32.491	0.648
14	0.440	4.432	41.900	47.653	0.636
15	0.431	4.504	41.100	52.63	0.741
16	0.110	1.241	14.400	17.493	0.243
17	0.053	0.450	7.6000	9.512	0.067
18	0.345	5.892	15.500	42.469	1.002
19	0.128	0.973	12.600	18.987	0.243
20	0.055	0.444	5.6000	7.546	0.153
21	0.057	0.508	5.7000	7.595	0.123
22	0.098	0.370	14.100	16.906	0.233
23	0.104	0.395	14.600	17.264	0.263
24	0.206	2.680	19.600	36.43	0.601
25	0.067	0.781	10.500	11.581	0.120
26	0.100	0.872	12.100	22.207	0.248
27	0.0106	1.757	12.700	20.67	0.253

The aim of this example is to obtain the proper reduction in the input (IT budget) and output (profit) such that the efficiency score of each bank branch is maintained greater than or equal to the previous values. As shown in Table 2, when we apply the proposed method the new efficiency score of banks are always greater than or equal to the efficiency scores before decreasing the values of IT budget and profit. It shows that this banking system is able to improving the operating efficiency of each bank branch.

Amirteimoori and Emrouznejad (2010) introduced a DEA-based method (hereinafter named as the “AE method”) to assess the efficiency of a set of DMUs after reducing the values of a given input and output. The purpose of AE is to preserve the efficiency scores of the DMUs after assigning the apt reduction to the input and output of all DMUs. In other words, the AE model maintains the efficiency score of each DMU calculated from the standard DEA (CCR or CRS) model before reducing input and output values.

Here we make a comparison between the results of the proposed method and the AE method. Table 2 displays the efficiency score ( $\bar{\theta}_j^{*AE}$ ) of the banks via the CCR DEA model (2) as well as the optimal solutions of  $\bar{c}_{ij}$  and  $\bar{p}_{rj}$  using the AE model. In addition, the renewal efficiency score ( $\bar{\theta}_j^{*newAE}$ ) of the banks is calculated using model (2) in the presence of  $\bar{c}_{ij}$  and  $\bar{p}_{rj}$ . As shown in Table 2, the reduction values of our model is almost similar to the AE method but our model is characterized by lower computational

complexity. The AE efficiency score before reduction ( $\bar{\theta}_j^{*AE}$ ) and after reduction ( $\bar{\theta}_j^{*newAE}$ ) for each DMU are exactly identical while in our method the efficiency score is improved in many units, excluding DMU<sub>13</sub>, DMU<sub>26</sub> and DMU<sub>27</sub>.

The AE method requires  $n$  repetitions of the multiplier CCR model (2) where every CCR model involves  $n+m+s+1$  constraints and  $m+s$  variables while we measure the efficiency scores of DMUs by solving single common-weights DEA model (7) consisting of  $n+m+s$  constraints and  $n+m+s$  variables. In this example  $n=27$ ,  $m=3$ ,  $s=2$ ,  $k=1$  and  $h=1$ , the AE method solves 27 models where every model has 33 constraints and 5 variables while we solve only one model with 32 constraints and 32 variables.

The AE model contains  $2n \times (k+h+1) + (n+1)(k+h)$  constraints where  $k$  and  $h$  are the numbers of reduction indexes for the inputs and outputs, respectively, whereas our proposed model (13) includes  $2n \times (k+h+1) + k+h+m+s$  constraints where  $m+s$  is corresponding with  $u_r, v_i \geq \varepsilon$ . Moreover, the AE method has  $2n \times (k+h) + n \times (k+h+1) + m+s$  variables, respectively, while our model contains  $m+s+3n(k+h)$  variables.

Table 2. The comparison of the results for the proposed method and the AE method

DMU <sub>j</sub>	AE method				Proposed method			
	$\bar{c}_{ij}$	$\bar{p}_{rj}$	$\bar{\theta}_j^{*AE}$	$\bar{\theta}_j^{*newAE}$	$\bar{c}_{ij}$	$\bar{p}_{rj}$	$\theta_j^*$	$\theta_j^{*new}$
1	0.0763	0.0971	0.721	0.721	0.0764	0.0928	0.568	0.579
2	0.0866	0.1423	0.792	0.792	0.0866	0.1392	0.614	0.632
3	0.1197	0.1519	0.634	0.634	0.1197	0.1546	0.471	0.474
4	0.1074	0.0883	0.662	0.662	0.1074	0.0851	0.452	0.456
5	0.0677	0.0992	0.632	0.632	0.0678	0.1005	0.477	0.506
6	0.2531	0.4617	0.763	0.763	0.2531	0.4640	0.761	0.772
7	0.0305	0.4617	1.000	1.000	0.0305	0.4640	0.973	1.000
8	0.0361	0.0833	0.555	0.555	0.0361	0.0851	0.538	0.545
9	0.7638	0.7777	0.625	0.625	0.7637	0.7811	0.597	0.599
10	0.0611	0.1147	0.505	0.505	0.0611	0.1005	0.499	0.500
11	0.0611	0.1147	0.503	0.503	0.0611	0.0851	0.498	0.502
12	0.0255	0.0741	0.669	0.669	0.0255	0.0770	0.662	0.683
13	0.1884	0.2712	0.949	0.949	0.1883	0.2706	0.800	0.800
14	0.2240	0.2662	0.591	0.591	0.2241	0.2628	0.580	0.586
15	0.2195	0.3102	0.670	0.670	0.2195	0.3094	0.652	0.663
16	0.0560	0.1017	0.676	0.676	0.0560	0.1005	0.666	0.666
17	0.0270	0.0280	0.718	0.718	0.0270	0.0308	0.706	0.708
18	0.1757	0.4194	1.000	1.000	0.1757	0.4156	1.000	1.000
19	0.0652	0.1017	0.840	0.840	0.0651	0.1005	0.787	0.797
20	0.0280	0.0640	0.999	0.999	0.0280	0.0618	0.713	0.725
21	0.0290	0.0515	0.774	0.774	0.0290	0.0541	0.694	0.733
22	0.0499	0.0975	0.807	0.807	0.0499	0.1005	0.695	0.716
23	0.0530	0.1101	0.862	0.862	0.0530	0.1082	0.684	0.688
24	0.1049	0.2516	0.954	0.954	0.1049	0.2470	0.932	0.951
25	0.0341	0.0502	0.627	0.627	0.0341	0.0464	0.626	0.630
26	0.0509	0.1038	1.000	1.000	0.0509	0.1546	1.000	1.000
27	0.0509	0.1059	1.000	1.000	0.0055	0.1082	1.000	1.000
Sum	3.0454	4.9997	20.528	20.528	3.0000	5.0000	18.645	18.911



In this example, the AE method includes 218 constraints, 194 variables while the proposed model contains 169 constraints, 172 variables. Obviously, the proposed model is computationally economical because of 49 constraints fewer than the AE method. Therefore, the proposed method decreases the number of constraints and variables, which leads to a strong reduction in the computational requirements.

Finally, the AE method re-solves  $n$  times the multiplier CCR model for measuring the efficiency of DMUs with consideration of the reductions, whereas our method re-solves only single common-weights DEA model. Indeed, the proposed method solves  $(n-1)$  linear programs fewer than the AE method and this constitutes a computational advantage.

Naturally, the reduction in the computational burden is primarily linked to the adoption of the common weights concept in all steps while the AE method uses different methods.

Briefly, Fig. 1 shows the above-mentioned comparisons between the AE method and the proposed method.

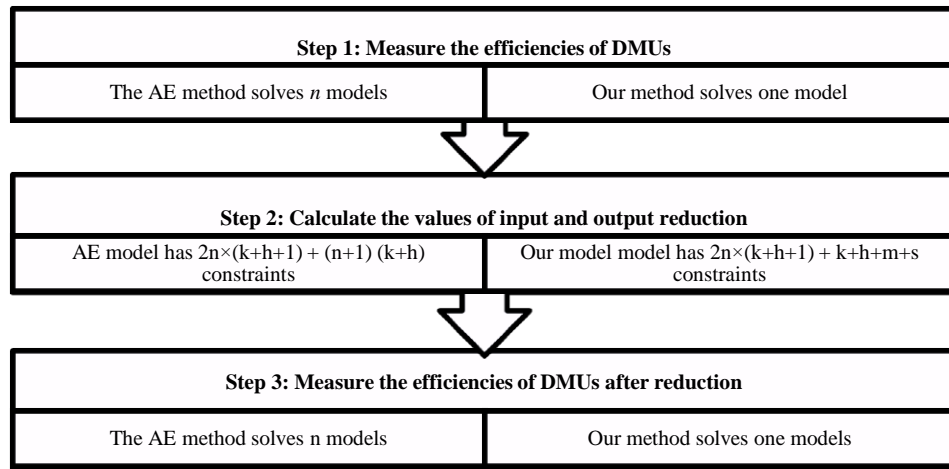


Fig. 1. Comparison between the AE and the proposed methods

## 6. Conclusions and future research directions

The integration of activity planning, resource allocation and performance managements are current challenges in both the theory and the practice of DEA. In this paper, we propose a new approach to improve the efficiency of the units when some given inputs and/or outputs are reduced in the evaluation process. Our aim is to optimize the resource contraction such that the efficiency of all DMUs will get bigger than or equal to the efficiency of previous change. In this paper, we first introduce a common weights method for measuring efficiency of DMUs before and after data change. Thus, we achieve the efficiencies by solving a linear program which is computationally economical. In addition, in comparison with total weights flexibility in the traditional DEA models, the common-weights DEA model takes into account the common weights. Then, based on the goal program (GP) concept we proposed a new method to find an adequate assignment for the reduction amount of inputs and outputs in the presence of the current data effect in the evaluation system. The proposed model is not only consistent with the

outlined managerial objectives; it also significantly reduces the computational burden for the analysis.

The developed framework in this paper can potentially lend itself to many practical applications. However, there are a number of challenges involved in the proposed research that provide a great deal of fruitful scope for future research. For example, there is no mechanism in the proposed method to determine the adequate reduction values for integer inputs and outputs. Another potential for future research is to identify the upper and lower bounds for  $C_i$  and  $P_r$ , respectively, such that the assignment system for reducing input/output remains feasible.

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